Evaluation of the B-Spline Equation for Wind Speed Extrapolation/Estimation

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1. BACKGROUND

BACKGROUND

IMPORTANCE OF WIND SPEED EXTRAPOLATION/ESTIMATION

- Thorough knowledge of wind speed is important for identifying suitable sites for wind projects, deciding on the type of wind turbine to be installed, wind turbine placement, electricity generation forecasting, and grid operation.
- Wind speed data are often measured using met masts as well as ground-based remote sensing devices such as light detection and ranging (LIDAR) and sonic detection and ranging (SODAR).
- Met masts typically conduct wind measurements below wind turbine hub heights because tall measurement towers are costly and are far more difficult to obtain approval for. Remote sensing technologies are standard for wind resource assessment in countries such as Germany [1]. Their bankability remains a challenge in South Africa [2] as lenders are hesitant to rely on wind resource data from remote sensing technologies, particularly due to concerns about the accuracy of such data in complex terrain [3].
- Accurate extrapolation models are important for estimating wind speeds from available lower heights to the steadily increasing hub heights of modern wind turbines

BACKGROUND

PRIMARY EQUATIONS FOR EXTRAPOLATING WIND SPEEDS

- The power law and the logarithmic (log) law are two primary mathematical models used to extrapolate wind speeds to heights of interest [4].
- The power law equation is shown below:

$$V_2 = V_1(\frac{H_2}{H_1})^{\alpha}$$

V_1 is the wind speed at a known height (H_1), and V_2 is the wind speed at a desired height (H_2). Alpha (α) is the wind shear exponent and refers to the wind speed changes with height due to numerous factors such as surface features, height, wind speed direction, and atmospheric conditions [5]. Alpha is calculated as shown below:

$$\alpha = \frac{ln(\frac{V_2}{V_1})}{ln(\frac{H_2}{H_1})}$$

- A high alpha value indicates a greater increase in wind speed with height. A lower value suggests a smaller increase. A negative value signifies that wind speed decreases with height.
- The power law equation is shown below:

$$V_2 = V_1 \frac{ln(\frac{H_2}{z_0})}{ln(\frac{H_1}{z_0})}$$

z_0 is the surface roughness length. The surface roughness length characterizes land cover within a region. Regions with high roughness length, such as dense forests, present more friction to wind than areas with low roughness length, such as bodies of water [6]. Julies (2021) provides roughness length values for various terrain types.

2. RESEARCH QUESTION

RESEARCH QUESTION

RESEARCH AIM AND QUESTION

- This study aimed to evaluate the performance of the quadratic B-spline method for extrapolating wind speed from lower elevations to turbine heights.
- The following question should be answered by this study:
    1. Does the quadratic B-spline method outperform the traditional wind speed extrapolation methods (power law and logarithmic law) when extrapolating wind speed from lower elevations to turbine heights?

WHY IT MATTERS FOR THE WIND INDUSTRY?

- Accurate extrapolation models are important for estimating wind speeds from measured lower heights to the increasing hub heights of modern wind turbines.
- Current research seeks to improve these models by assessing alternative equations for wind speed extrapolation.

3. METHODOLOGY

METHODOLOGY

STUDY LOCATION AND COLLECTION OF DATA

Anemometer	Height (m)
1st anemometer	60 m
2nd anemometer	80 m
3rd anemometer	100 m

MET MAST A

- **Location**: Eastern Cape
- **Data Period**: Jan 2023– Dec 2024
- **Elevation**: 1543 m above sea level
- **Terrain**: Open terrain (z_0 = 0.05 m), surrounded by small areas of agricultural land.
- Wind speed extrapolated to 120 m and compared with actual wind speed measurements from an operational wind farm with a hub height of 120 m.

Anemometer	Height (m)
1st anemometer	14 m
2nd anemometer	40 m
3rd anemometer	47 m
4th anemometer	60 m

MET MAST B

- **Location**: Northern Cape
- Data Period: Jan 2012– Dec 2013
- **Elevation**: 1681 m above sea level
- **Terrain**: Open terrain (z_0 = 0.05 m), surrounded by dense low shrubs.
- Wind speed extrapolated to 60 m and compared with actual anemometer measurements at 60 m.

Anemometer	Height (m)
1st anemometer	30 m
2nd anemometer	60 m
3rd anemometer	90 m
4th anemometer	120 m

MET MAST C

- **Location**: Western Cape
- Data Period: Aug 2023– Aug 2025
- **Elevation**: 1255 m above sea level
- **Terrain**: Open terrain (z_0 = 0.05 m), surrounded by scattered low shrubs.
- Wind speed extrapolated to 120 m and compared with actual anemometer measurements at 120 m.

METHODOLOGY

PROCESSING/CLEANING WIND DATA

- Processing raw wind speed data is essential, as raw data is not sufficiently reliable for direct use in wind resource assessment.
- The met mast data contained several types of invalid data, including duplicate timestamps, erroneous wind speed measurements, missing data, tower shading, and flatline data. The wind farm dataset consisted of duplicate timestamps and missing data.
- Tower shading is the distortion in wind speed and direction measurements caused by the met mast structure physically blocking or disturbing airflow before it reaches a sensor [8].
- Flatline data was defined as wind speed values that change less than 0.25 m/s over a two hour period.

- The steps below outline the process of cleaning the raw wind data:
  - **Step 1**: The duplicate timestamps were removed in Excel.
  - **Step 2**: The data was then transferred to Windographer (version 6.1.0) to identify the remaining invalid data.
  - **Step 3**: The invalid data were removed and filled in with reference data using the linear least squares algorithm. Reference data sources included co-located sensors (first option), a nearby third-party met mast dataset (second option), and long-term datasets from ERA5 and MERRA-2 (third option), with correlations of at least 0.7.

METHODOLOGY

B-SPLINE CURVE MODEL

- B-splines were formulated to provide a flexible and mathematically robust framework for curve representation [9]. In simple terms, the B-spline curve is a method for constructing a smooth, continuous curve through a defined set of control points.

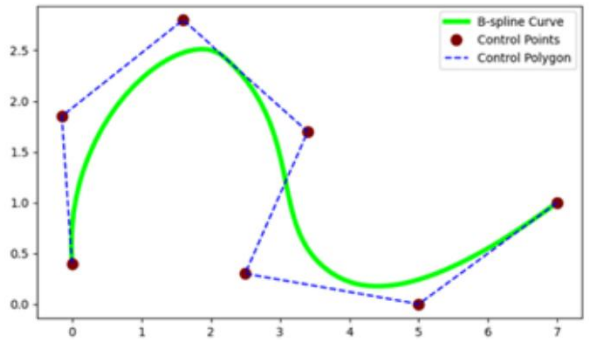


Figure 1: A B-spline curve defined through a set of control points. Source: [10].

- The mathematical representation of a B-spline curve model is given by the following equation:

$$S(u) = \sum_{i=1}^{n} B_{i,p}(u) P_i$$

Where $S(u)$ denotes the extrapolated wind speed at a normalized height (u).

- The normalized height is calculated using the formula below:

$$u = \frac{y - y_{minimum}}{y_{maximum} - y_{minimum}}$$

  - y is the height of interest
  - $y_{minimum}$ denotes the height of the minimum control point
  - $y_{maximum}$ is the height of the maximum control point
  - P_i represents the control point which governs the shape of the curve. P_i was assumed to be the overall average wind speed measured by the sensors at a specific height
  - $B_{i,p}$ is the basis function of degree p associated with a control point
  - i is the identification number of the control point
  - n is the total number of control points.
- The B-spline equation with three control points, also known as the quadratic B-spline, presented below, was used for extrapolation as three control points were available at each site.

$$S(u) = (1-u)^2 P_1 + 2u(1-u)P_2 + (u)^2 P_3$$

METHODOLOGY

WHY THE B-SPLINE CURVE MODEL?

- The quadratic B-spline equation was selected to extrapolate wind speeds to turbine heights for three reasons:
    i. Three control points for each site were used to estimate wind speeds to turbine height
    ii. The equation produces a curved shape which is characteristic of a typical wind speed profile (logarithmic profile), as shown below:

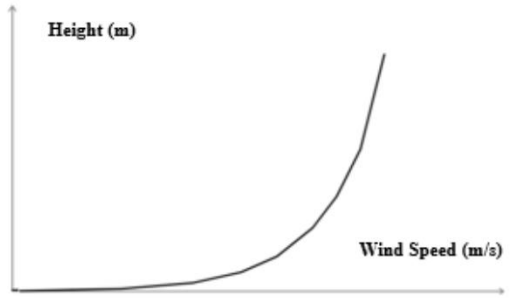


Figure 2: A wind profile where wind speed is increasing with height. Source: [11].

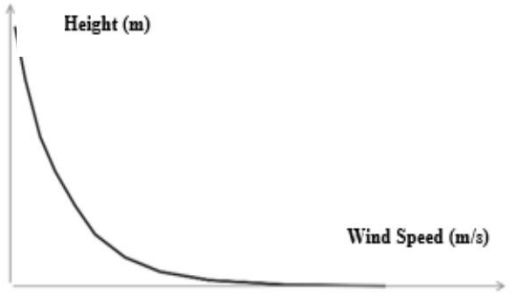


Figure 3: A wind profile where wind speed is increasing with height. Source: [11].

    iii. The equation can show an increase or decrease in wind speed with height, and such patterns are reflected in the quadratic B-spline curve.

WIND DATA CLASSIFICATION

- To assess changes in wind speed over time, the wind-speed measurements were separated into daytime and nighttime.
- Thereafter, the day and night measurements were analyzed to identify the occurrence of inversion.
- Inversion at a site was identified when:

$$P_1 \geq P_2 \text{ or } P_2 \geq P_3$$

- The variation of wind speed with height was expressed as the natural logarithm of the ratio of two control points:

$$\text{Wind speed change with height} = ln(\frac{P_2}{P_1})$$

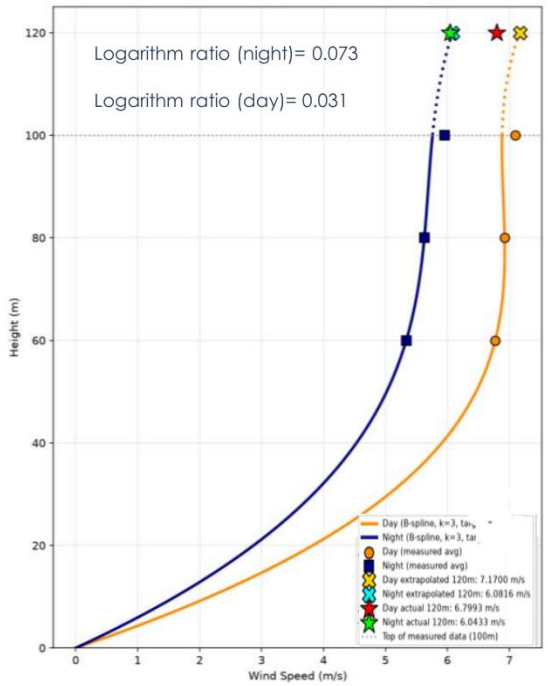
- A higher value indicated a greater increase in wind speed with height. A lower value indicated a smaller increase. A negative value indicated that wind speed decreases with height.

4. KEY FINDINGS

Extrapolation Method	Night (m/s)	Night Acc. (%)	Day (m/s)	Day Acc. (%)
Actual wind speed at 120 m	6.043	-	6.799	-
Quadratic B-spline at 120 m	6.082	100.63	7.170	105.45
Power law at 120 m	6.191	102.44	7.221	106.21
Log law at 120 m	6.094	100.84	7.269	106.91

SITE A: EXTRAPOLATION FROM 100 M TO 120 M

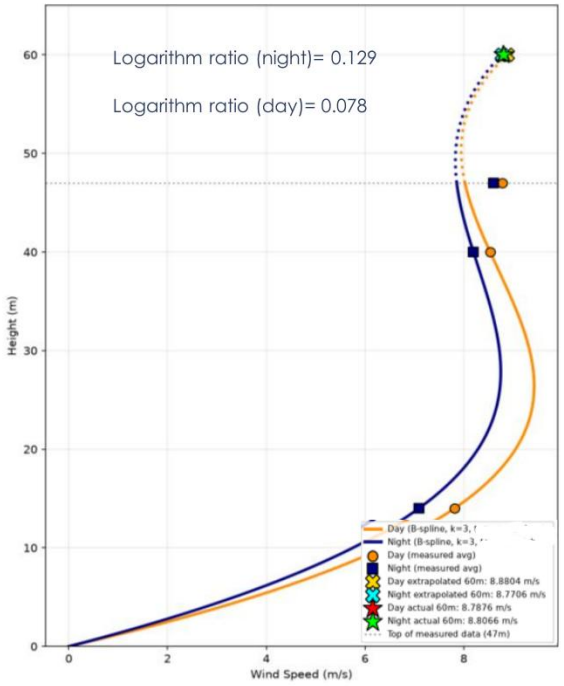
The B-spline model outperformed the other two extrapolation methods for both day and night.



Extrapolation Method	Night (m/s)	Night Acc. (%)	Day (m/s)	Day Acc. (%)
Actual wind speed at 60 m	8.807	-	8.788	-
Quadratic B-spline at 60 m	8.771	99.59	8.880	101.06
Power law at 60 m	9.028	102.51	9.040	102.87
Log law at 60 m	8.907	101.14	9.094	103.49

SITE B: EXTRAPOLATION FROM 47 M TO 60 M

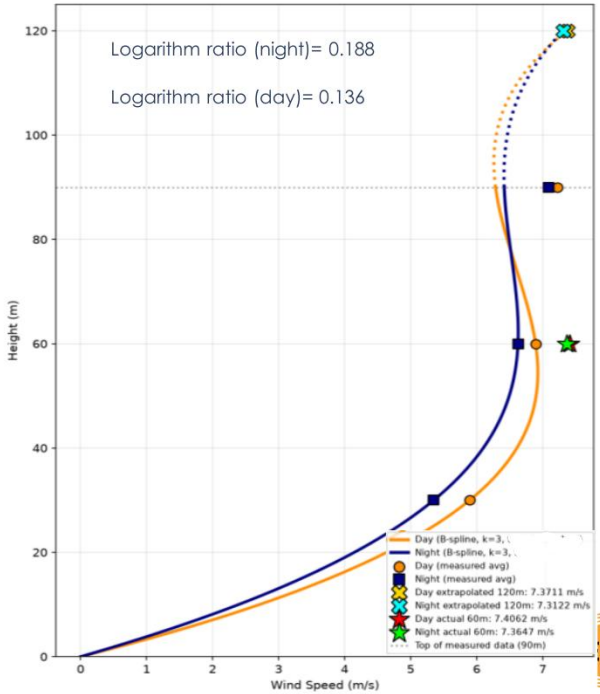
The B-spline model outperformed the other two extrapolation methods for both day and night.



Extrapolation Method	Night (m/s)	Night Acc. (%)	Day (m/s)	Day Acc. (%)
Actual wind speed at 120 m	7.365	-	7.406	-
Quadratic B-spline at 120 m	7.364	99.99	7.371	99.53
Power law at 120 m	7.608	103.30	7.598	102.59
Log law at 120 m	7.312	99.29	7.501	101.28

SITE C: EXTRAPOLATION FROM 90 M TO 120 M

The B-spline model outperformed the other two extrapolation methods for both day and night.



SAWEA

W SETA

Extrapolation Method	Night (m/s)	Night Acc. (%)	Day (m/s)	Day Acc. (%)
Actual wind speed at 120 m	6.043	-	6.799	-
Quadratic B-spline at 120 m	5.822	96.34	6.718	98.81
Power law at 120 m	6.756	111.79	9.251	136.06
Log law at 120 m	5.938	98.26	7.304	107.43

SITE A: EXTRAPOLATION FROM 80 M TO 120 M

- MERRA-2 wind speed at 50 m was added as a supplementary control point.
- The B-spline model outperformed the power law and log models during the day. The model ranked second during the night, after the log law model.

Extrapolation Method	Night (m/s)	Night Acc. (%)	Day (m/s)	Day Acc. (%)
Actual wind speed at 60 m	8.807	-	8.788	-
Quadratic B-spline at 60 m	9.023	102.46	8.767	99.76
Power law at 60 m	10.20	115.82	10.743	122.25
Log law at 60 m	8.693	98.71	9.053	103.01

SITE B: EXTRAPOLATION FROM 40 M TO 60 M

- MERRA-2 wind speed at 10 m was added as a supplementary control point.
- The B-spline model outperformed the power law and log models during the day. The model ranked second during the night after, the log law model.

Extrapolation Method	Night (m/s)	Night Acc. (%)	Day (m/s)	Day Acc. (%)
Actual wind speed at 120 m	7.365	-	7.406	-
Quadratic B-spline at 120 m	7.608	103.30	7.566	102.16
Power law at 120 m	9.925	134.76	8.904	120.22
Log law at 120 m	7.274	102.16	7.881	106.41

SITE C: EXTRAPOLATION FROM 60 M TO 120 M

- MERRA-2 wind speed at 10 m was added as a supplementary control point.
- The B-spline model outperformed the power law and log models during the day. The model ranked second during the night, after the log law model.

Extrapolation Method	Night (m/s)	Night Acc. (%)	Day (m/s)	Day Acc. (%)
Actual wind speed at 120 m	6.043	-	6.799	-
Quadratic B-spline at 120 m	5.512	91.20	10.118	148.81
Power law at 120 m	7.459	123.43	10.680	157.07
Log law at 120 m	5.858	96.93	7.435	109.34

SITE A: EXTRAPOLATION FROM 60 M TO 120 M

- ERA5 wind speed at 10 m was added as a supplementary control point.
- The B-spline model ranked second for both day and night, after the log law model.

SITE B: EXTRAPOLATION FROM 14 M TO 60 M

- Wind speeds at the first height (14 m) were not estimated because a third control point below 10 m was unavailable for inclusion in the quadratic B-spline equation.

SITE C: EXTRAPOLATION FROM 30 M TO 120 M

- Wind speeds at the first height (30 m) were not estimated because a third control point below 10 m was unavailable for inclusion in the quadratic B-spline equation.

5. LIMITATIONS AND NEXT STEPS

LIMITATIONS AND NEXT STEPS

CONCLUSION

- The results showed that for sites with a roughness length of 0.05 m, the quadratic B-spline method performed best in estimating wind speeds at higher elevations when all the control points were derived from measured data.
- Developers assessing sites with a roughness length lower or higher than 0.05 m should re-validate the quadratic B-spline method locally before adopting it.
- The B-spline method became less accurate when ERA5 and MERRA-2 were added as supplementary control points.

LIMITATIONS

- A limitation of the quadratic B-spline equation is that it requires three measurement heights, unlike the power law and log law models, which require one measurement height. As such, long term datasets (ERA5 and MERRA-2) were added as supplementary control points when extrapolating wind speeds from the first and second heights.
- Since the quadratic B-spline method performed best when the control points were derived from measured data, wind developers should prioritize reliable wind speed when utilizing the quadratic B-spline approach. Long term data sets, such as ERA5 and MERRA-2, are recommended for filling in missing data and replacing invalid data.

NEXT STEPS

- To improve the performance of the tested B-spline model, the study suggests extrapolating wind speeds at the first and second heights using control points derived from measured data.
- Future steps in this research domain could include testing the quadratic B-spline method on more diverse terrains to expand the model's applicability and reliability.

6. REFERENCES

REFERENCES

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QUESTIONS